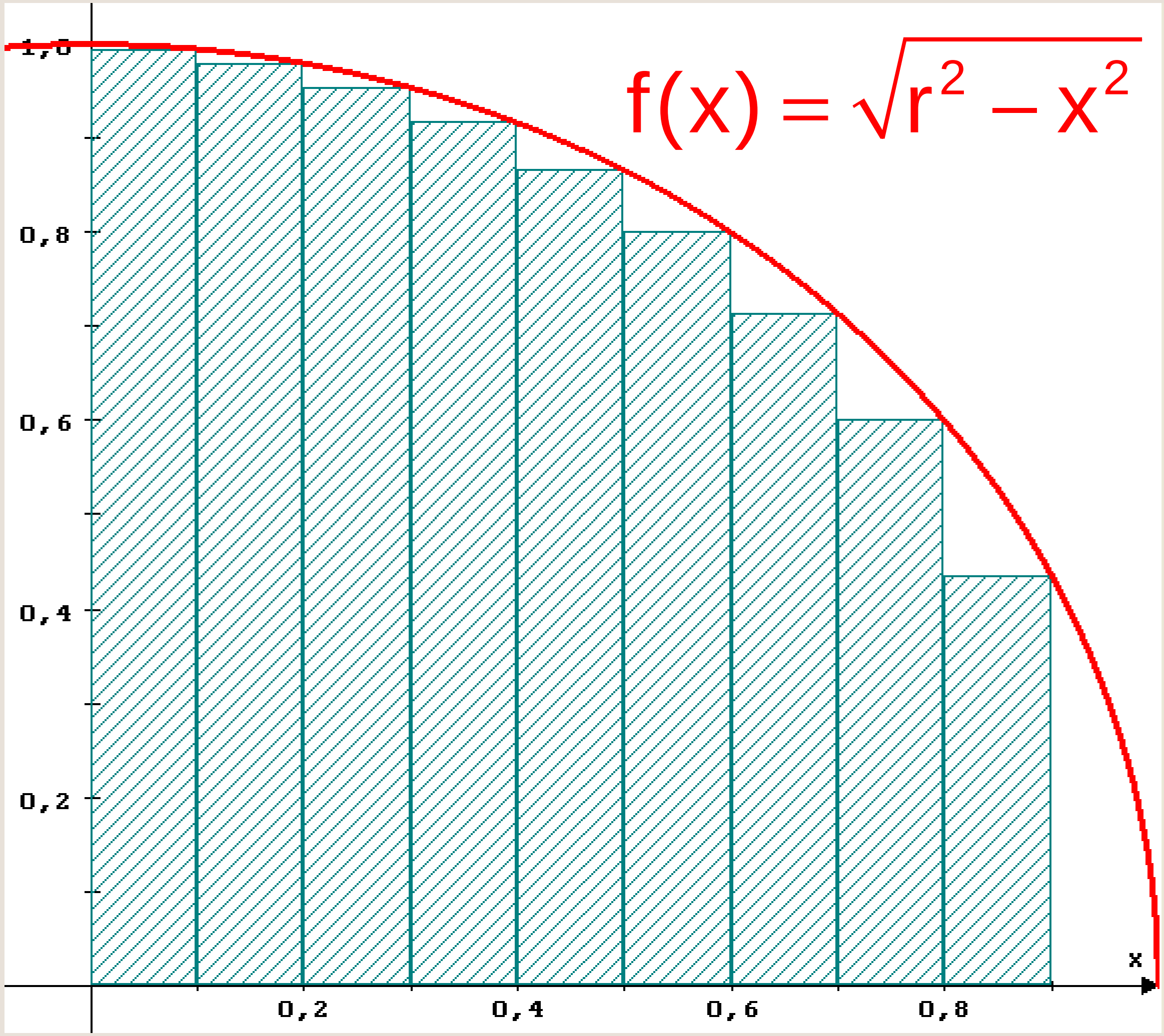
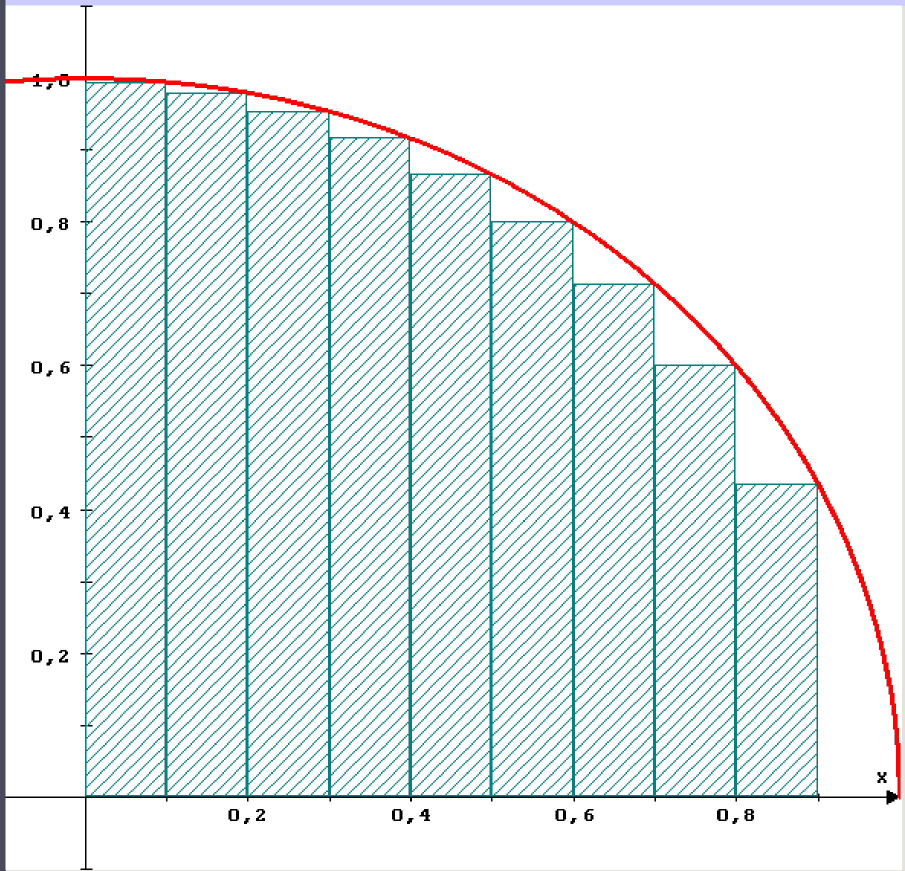
Flächeninhalt des Kreises

Rechteckmethode



Flächeninhalt des Kreises



$$U_{10} = \frac{r}{10}\left[\sqrt{r^2 - \left[1 \cdot \frac{r}{10}\right]^2} + \sqrt{r^2 - \left[2 \cdot \frac{r}{10}\right]^2} + \ldots\ldots. + \sqrt{r^2 - \left[9 \cdot \frac{r}{10}\right]^2}\right]$$

Flächeninhalt des Kreises

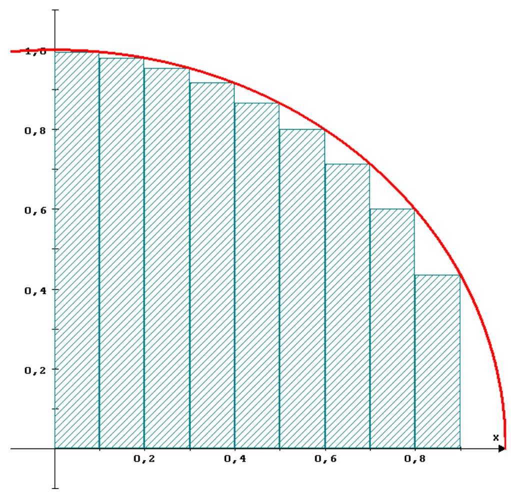
$$U_{10} = \frac{r}{10}\left[\sqrt{r^2 - \left[1\cdot\frac{r}{10}\right]^2} + \sqrt{r^2 - \left[2\cdot\frac{r}{10}\right]^2} + \ldots\ldots\ldots + \sqrt{r^2 - \left[9\cdot\frac{r}{10}\right]^2}\right]$$

$$= \frac{r^2}{10}\left[\sqrt{1-\frac{1^2}{10^2}} + \sqrt{1-\frac{2^2}{10^2}} + \ldots\ldots\ldots + \sqrt{1-\frac{9^2}{10^2}}\right]$$

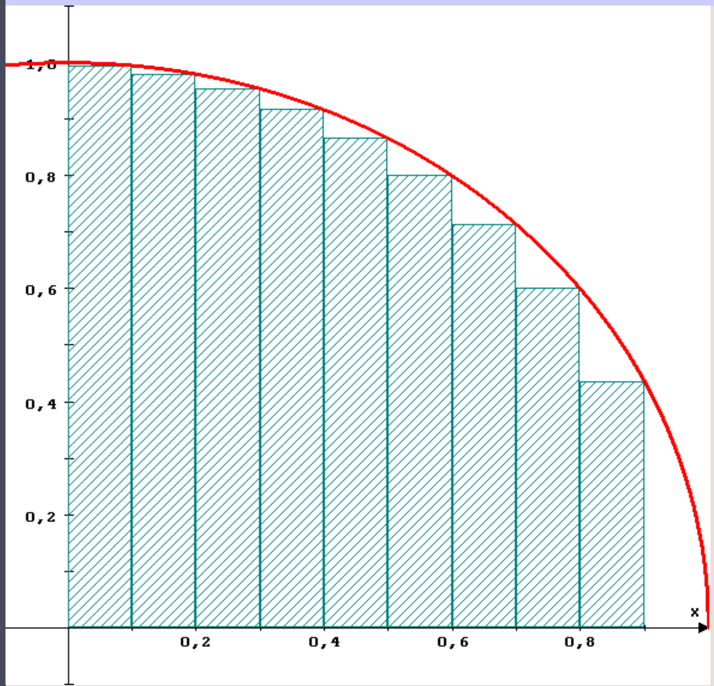
$$= \frac{r^2}{10^2}\left[\sqrt{10^2-1^2} + \sqrt{10^2-2^2} + \ldots\ldots\ldots + \sqrt{10^2-9^2}\right]$$

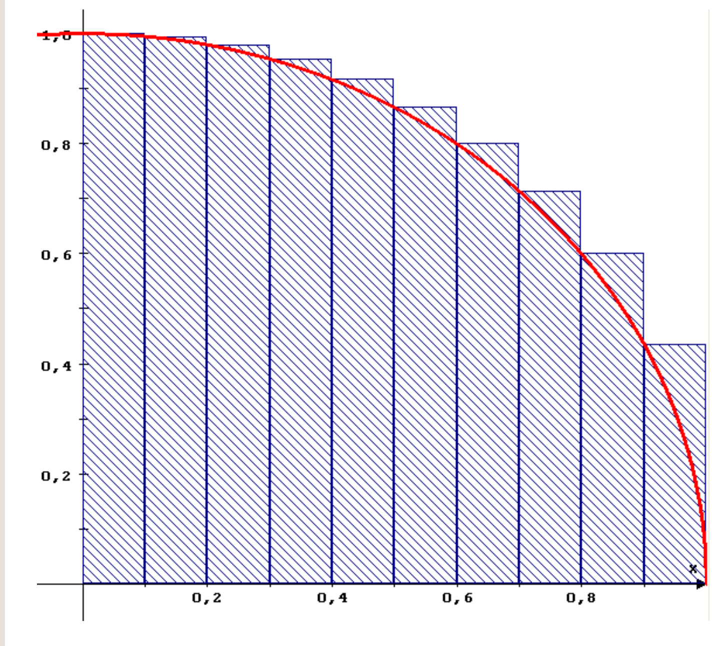
$$U_{10} = \frac{r^2}{10^2}\cdot\sum_{i=1}^{9}\sqrt{10^2-i^2}$$

$$U_n = \frac{r^2}{n^2}\cdot\sum_{i=1}^{n-1}\sqrt{n^2-i^2}$$



Flächeninhalt des Kreises

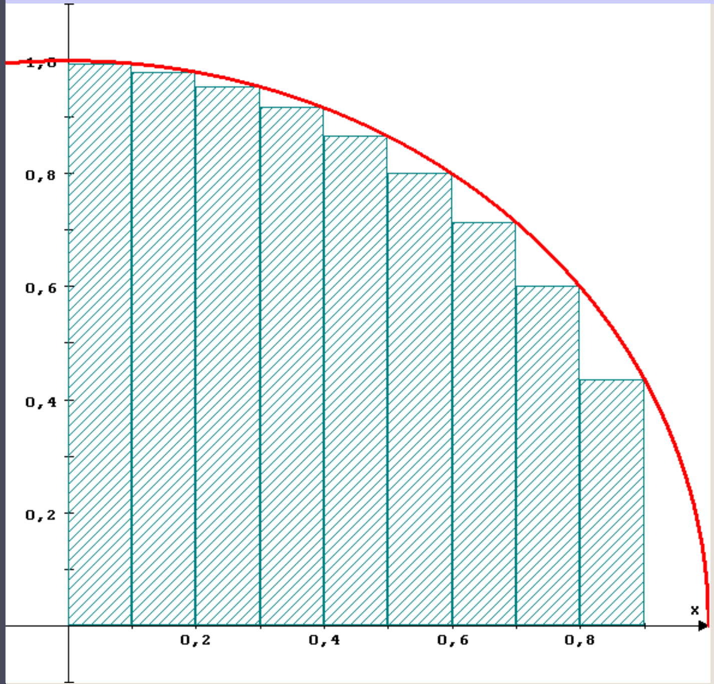


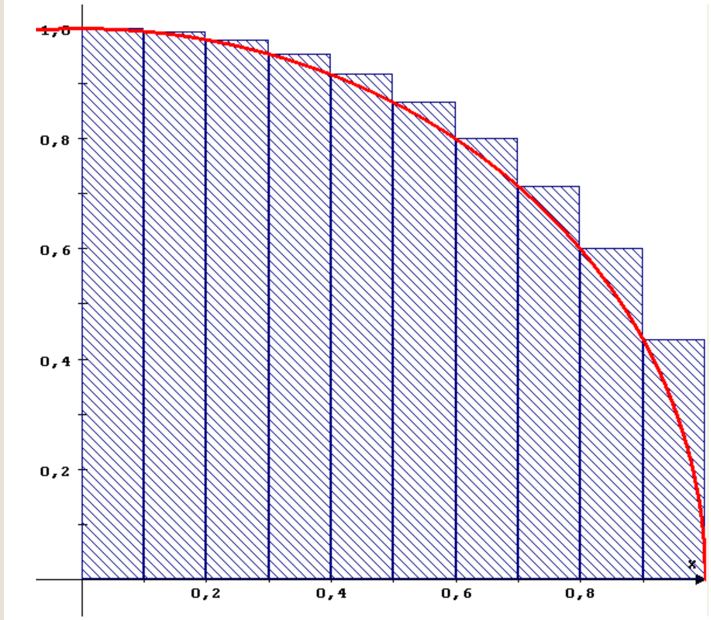


$$O_{10} = U_n + \frac{r}{10}\sqrt{r^2} = U_n + \frac{r^2}{10^2}\sqrt{10^2 - 0^2}$$

$$\Rightarrow \quad O_{10} = \frac{r^2}{10^2} \cdot \sum_{i=0}^{10} \sqrt{10^2 - i^2}$$

Flächeninhalt des Kreises

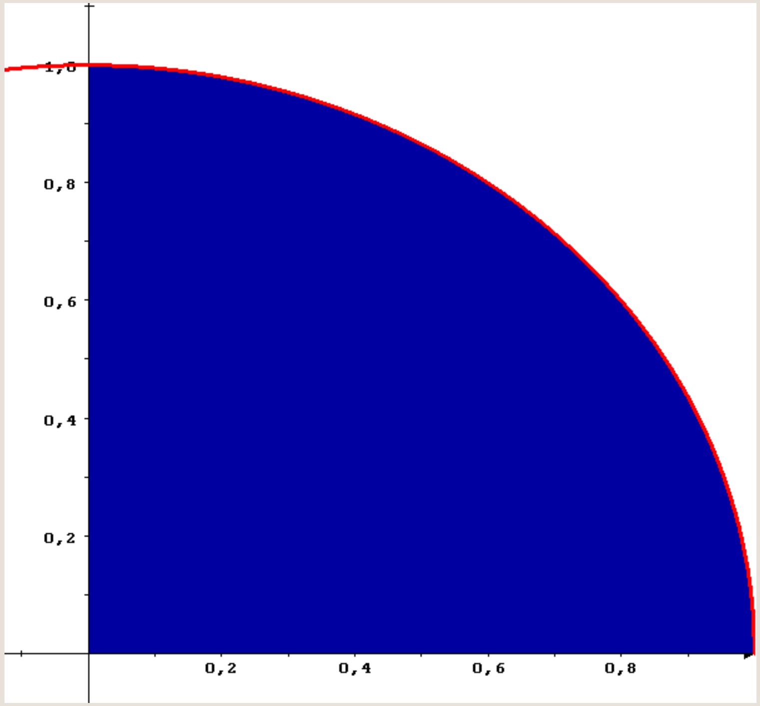




$$O_n = U_n + \frac{r}{n}\sqrt{r^2} = U_n + \frac{r^2}{n^2}\sqrt{n^2 - 0^2}$$

$$\Rightarrow \quad O_n = \frac{r^2}{n^2} \cdot \sum_{i=0}^{10} \sqrt{n^2 - i^2}$$

Flächeninhalt des Kreises



$$U_{10} < A < O_{10}$$

$$U_n < A < O_n$$

Algebra 1

#1: $\frac{\pi}{4}$

#2: 0.7853981633

#3: n := 10000

#4: $\frac{r^2}{n^2} \cdot \sum_{i=1}^{n-1} \sqrt{(n^2 - i^2)}$

n=10

#5: $0.7261295815 \cdot r^2$

n=100

#6: $0.7801042579 \cdot r^2$

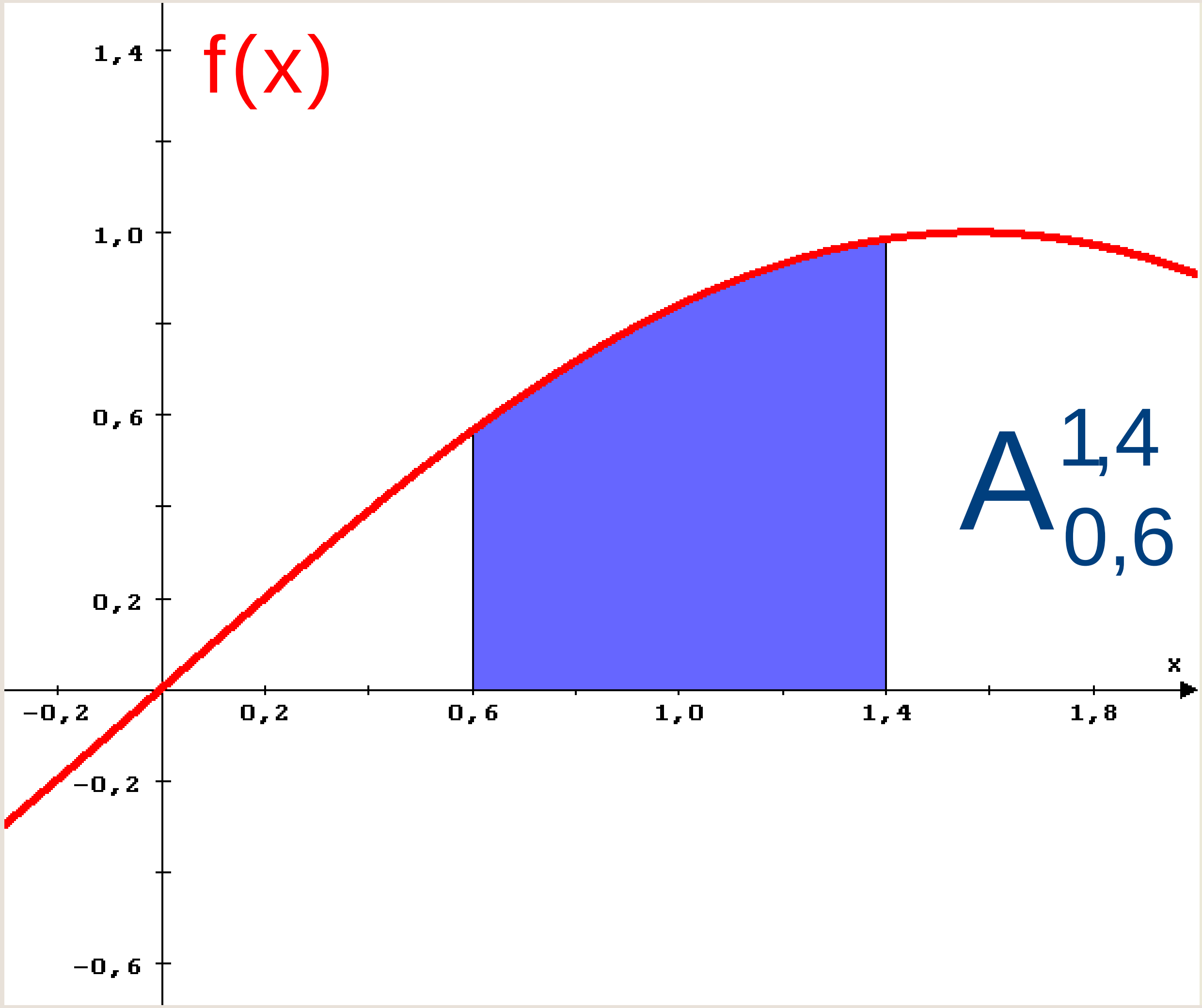
n=1000

#7: $0.7848888667 \cdot r^2$

n=10000

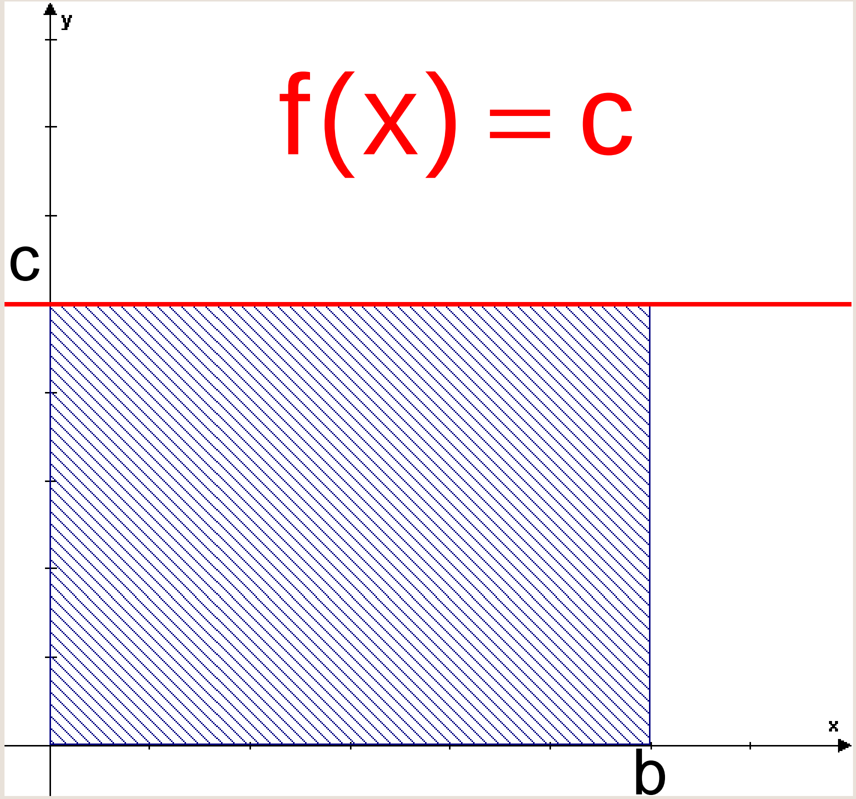
#8: $0.7853478694 \cdot r^2$

Flächen unterhalb von Funktionsgraphen



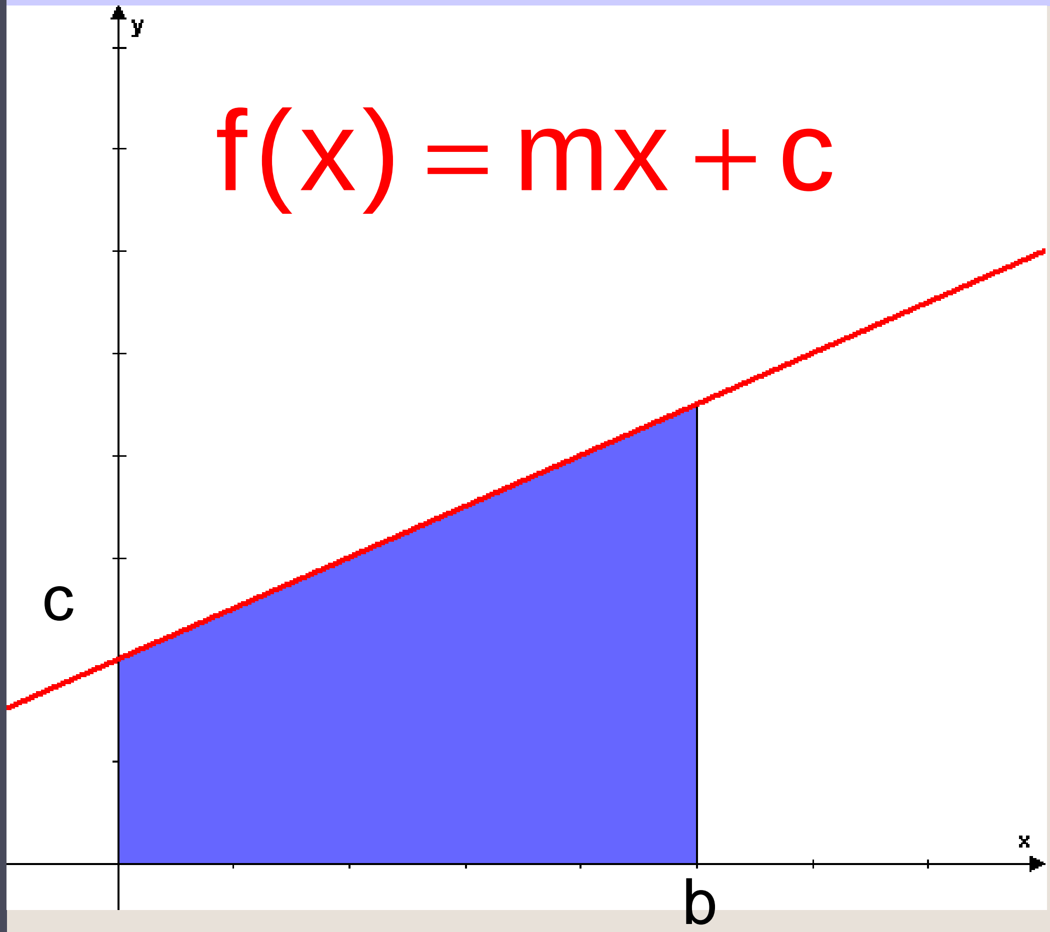
$$A_{0,6}^{1,4} = ?$$

Konstante Funktion



$$A_0^b = b \cdot c$$

Lineare Funktion

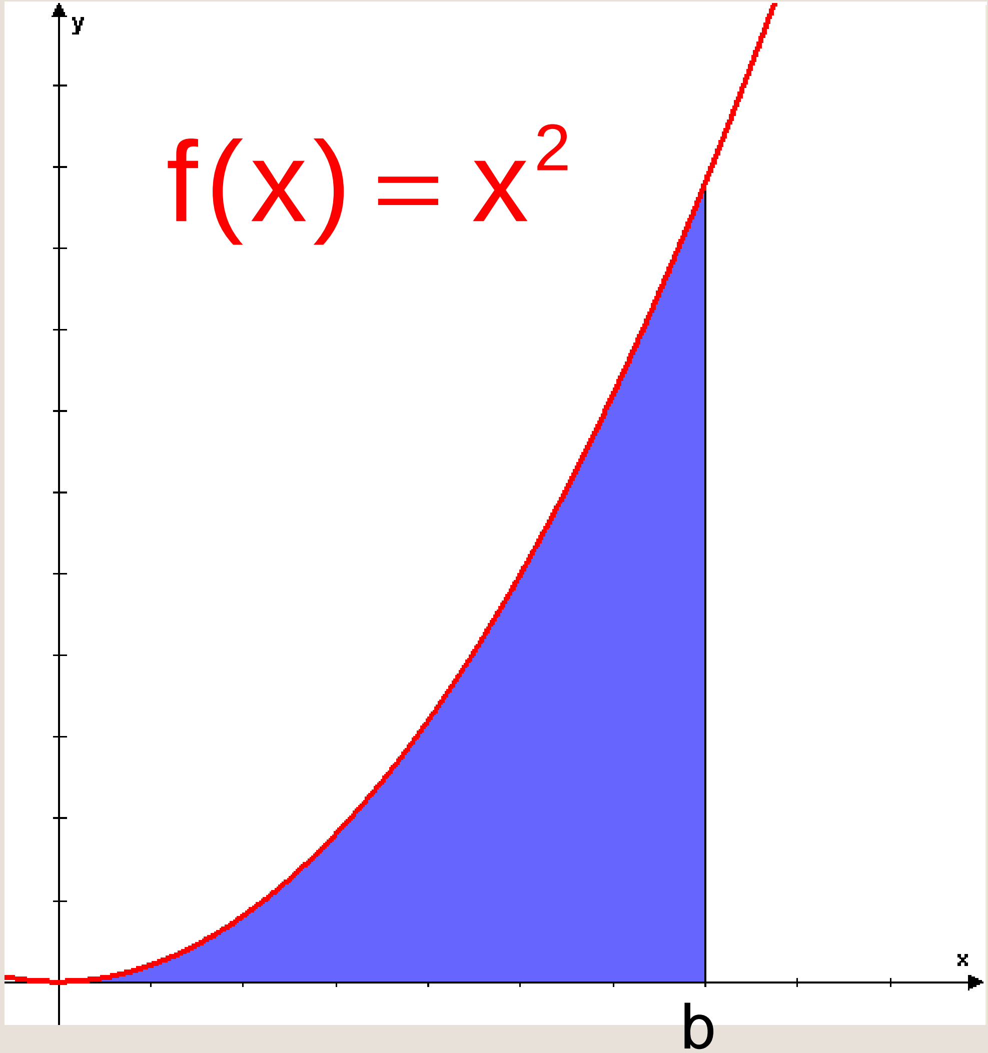


$$A_o^b = \frac{f(0) + f(b)}{2} \cdot b$$

$$A_o^b = \frac{0 + mb + c}{2} \cdot b$$

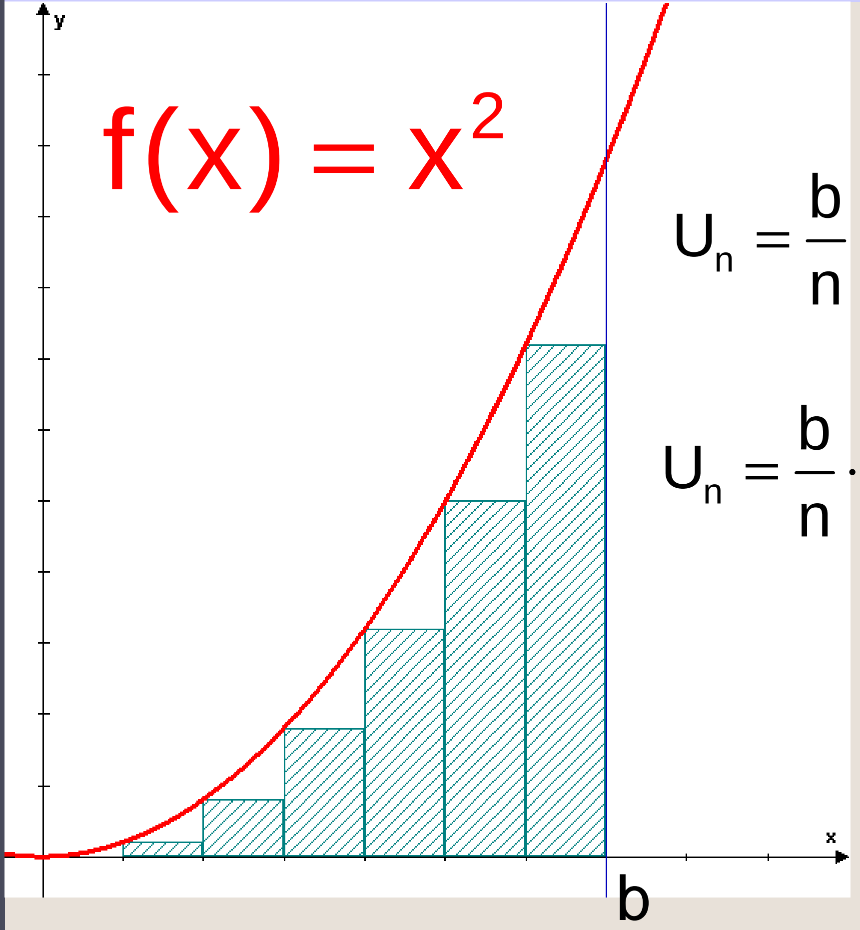
$$= \frac{m}{2} b^2 + cb$$

Quadratische Funktion



$$A_o^b = ?$$

Quadratische Funktion



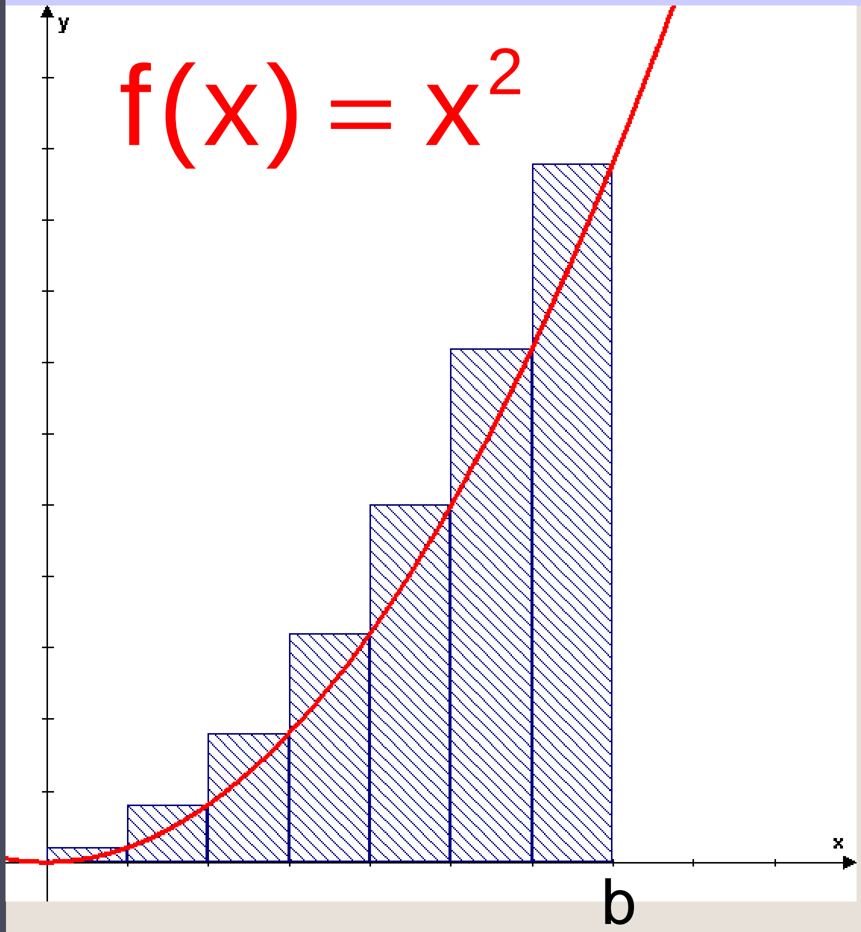
$$f(x) = x^2$$

$$U_n = \frac{b}{n} \cdot \left[f(1 \cdot \frac{b}{n}) + f(2 \cdot \frac{b}{n}) + \ldots + f((n-1) \cdot \frac{b}{n}) \right]$$

$$U_n = \frac{b}{n} \cdot \left[(1 \cdot \frac{b}{n})^2 + (2 \cdot \frac{b}{n})^2 + \ldots + ((n-1) \cdot \frac{b}{n})^2 \right]$$

$$U_n = \frac{b^3}{n^3} \cdot \left[1^2 + 2^2 + \ldots + (n-1)^2 \right]$$

Quadratische Funktion



$$U_n = \frac{b^3}{n^3} \cdot \left[1^2 + 2^2 + \ldots + (n-1)^2\right]$$

$$O_n = U_n + \frac{b}{n} \cdot b^2 = U_n + \frac{b^3}{n^3} \cdot n^2$$

$$O_n = \frac{b^3}{n^3} \cdot \left[1^2 + 2^2 + \ldots + (n-1)^2 + n^2\right]$$

Die Summe der ersten n Quadratzahlen

Algebra 1

#1: $\sum_{i=1}^{n-1} i^2$

#2: $\frac{n^3}{3} - \frac{n^2}{2} + \frac{n}{6}$

#3: $\sum_{i=1}^{n} i^2$

#4: $\frac{n \cdot (n + 1) \cdot (2 \cdot n + 1)}{6}$

Quadratische Funktion

$$U_n = \frac{b^3}{n^3} \cdot \left[1^2 + 2^2 + ... + (n-1)^2\right]$$

$$U_n = \frac{b^3}{n^3} \cdot \frac{(n-1)n(2n-1)}{6} = \frac{b^3}{n^3} \cdot \frac{2n^3 - 3n^2 + n}{6}$$

$$= \frac{b^3}{6} \cdot \frac{2n^3 - 3n^2 + n}{n^3} = \frac{b^3}{6} \cdot \left[2 - \frac{3}{n} + \frac{1}{n^2}\right]$$

$$O_n = \frac{b^3}{n^3} \cdot \left[1^2 + 2^2 + ... + (n-1)^2 + n^2\right]$$

$$O_n = \frac{b^3}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} = \frac{b^3}{n^3} \cdot \frac{2n^3 + 3n^2 + n}{6}$$

$$= \frac{b^3}{6} \cdot \frac{2n^3 + 3n^2 + 2n}{n^3} = \frac{b^3}{6} \cdot \left[2 + \frac{3}{n} + \frac{2}{n^2}\right]$$

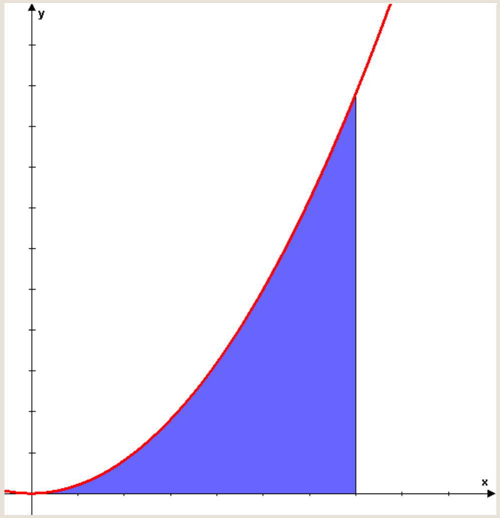
Quadratische Funktion

$$U_n = \frac{b^3}{6} \cdot \left[2 - \frac{3}{n} + \frac{1}{n^2}\right]$$

$$O_n = \frac{b^3}{6} \cdot \left[2 + \frac{3}{n} + \frac{2}{n^2}\right]$$

$$\lim_{n\to\infty} U_n = \frac{b^3}{6} \cdot 2 = \frac{b^3}{3}$$

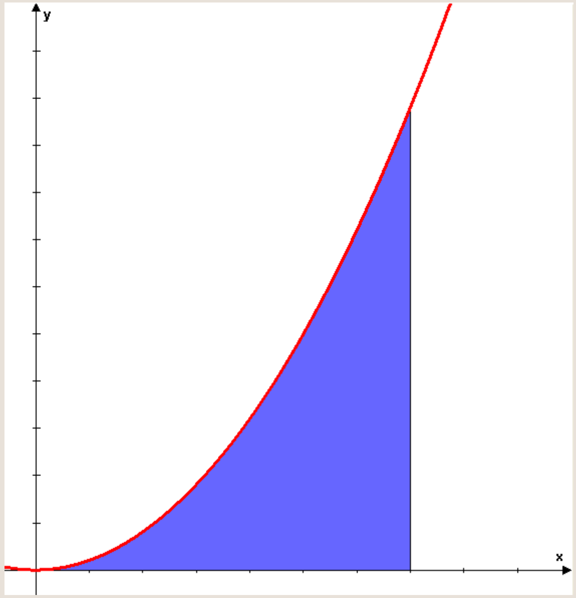
$$\lim_{n\to\infty} O_n = \frac{b^3}{6} \cdot 2 = \frac{b^3}{3}$$



$$A_o^b = \frac{1}{3} b^3$$

Definition des Flächeninhalts (1)

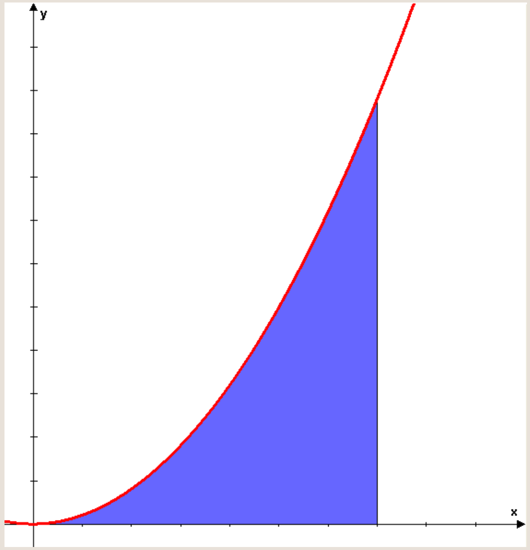
1. Fall $f(x) \geq 0$ für alle $x \in [0;b]$



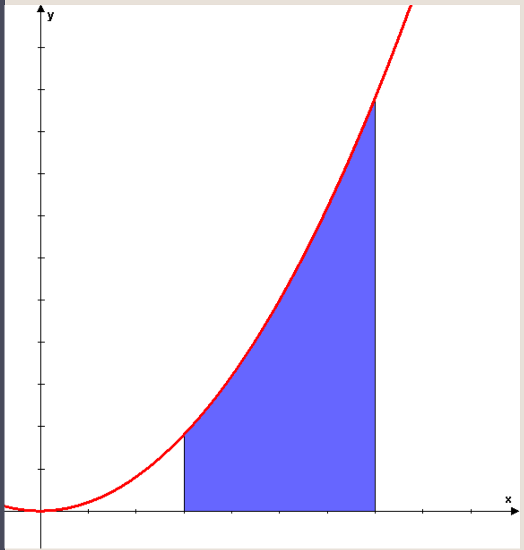
Wenn für eine Funktion f die Grenzwerte der Obersumme und der Untersumme übereinstimmen, dann wird durch diesen gemeinsamen Grenzwert der Inhalt der Fläche zwischen dem Graphen und der x-Achse im Bereich von 0 bis b definiert.

$$\lim_{n\to\infty} U_n = I \ \wedge \ \lim_{n\to\infty} O_n = I \ \Rightarrow A_o^b = I$$

Quadratische Funktion

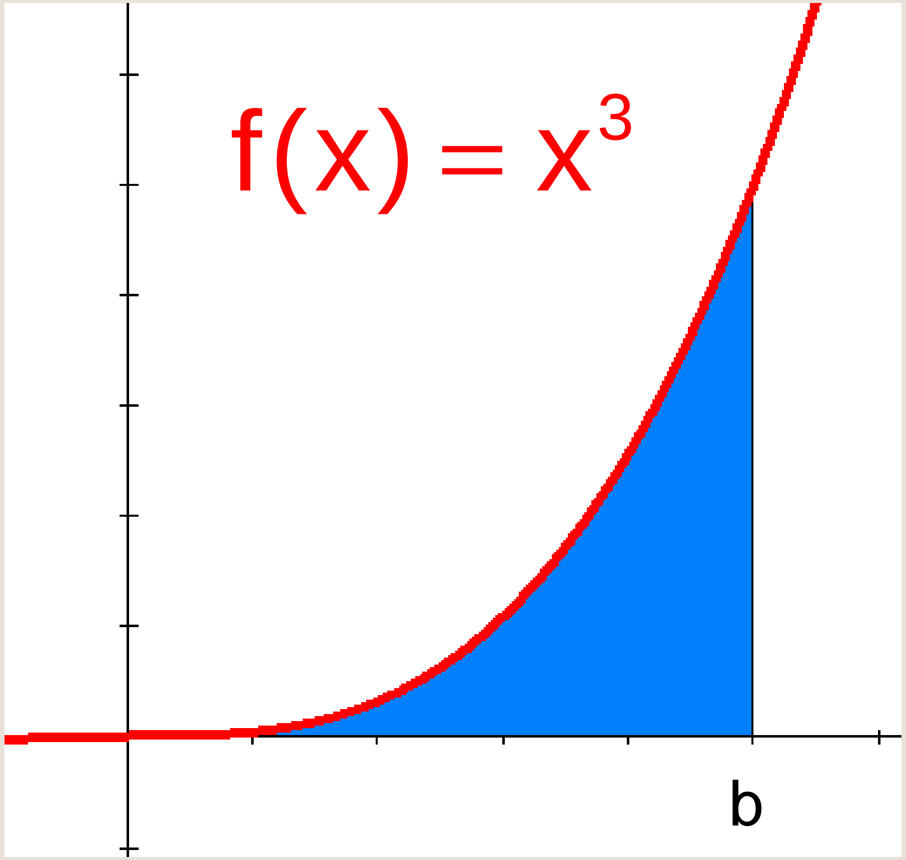


$$A_o^b = \frac{1}{3}b^3$$



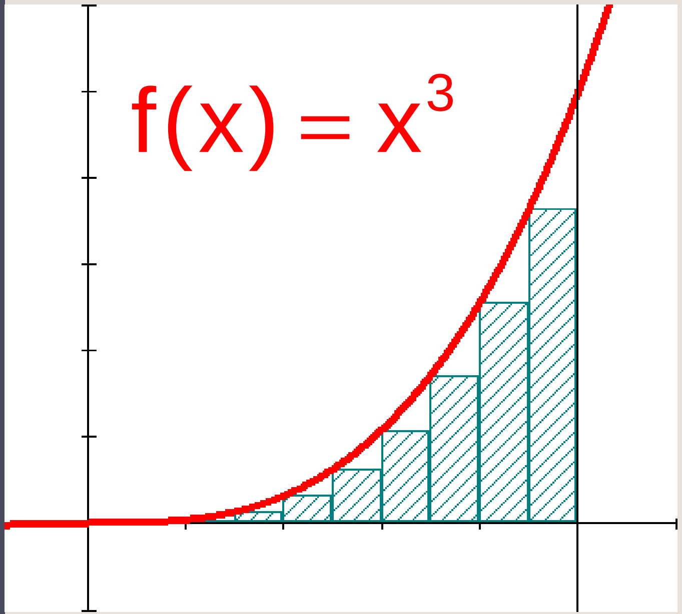
$$A_a^b = \frac{1}{3}b^3 - \frac{1}{3}a^3$$

Kubische Funktion



$A_o^b = ?$

Kubische Funktion



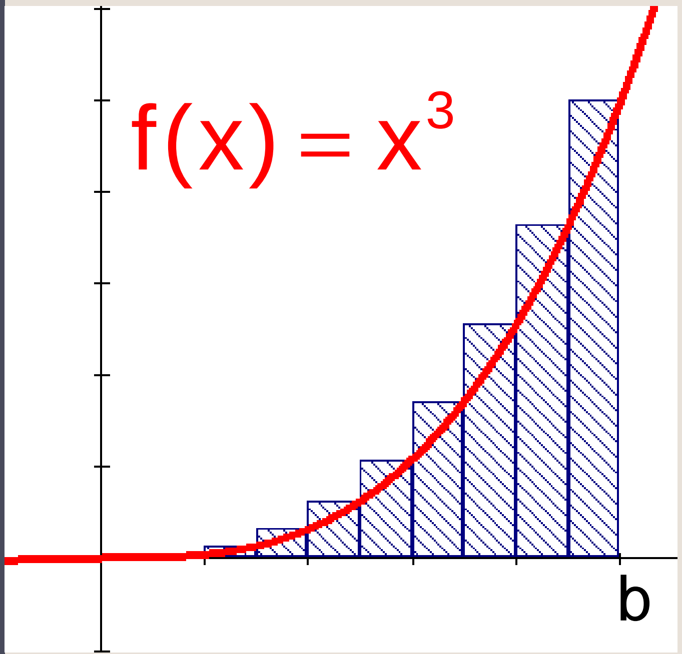
$$U_n = \frac{b}{n} \cdot \left[f(1 \cdot \frac{b}{n}) + f(2 \cdot \frac{b}{n}) + \ldots + f((n-1) \cdot \frac{b}{n}) \right]$$

$$U_n = \frac{b^4}{n^4} \cdot \left[1^3 + 2^3 + \ldots + (n-1)^3 \right]$$

$$1^3 + 2^3 + \ldots + n^3 = \left(\frac{n \cdot (n+1)}{2} \right)^2 \Rightarrow U_n = \frac{b^4}{n^4} \cdot \left[\frac{(n-1)^2 \cdot n^2}{4} \right] = \frac{b^4}{4} \cdot \left[1 - \frac{2}{n} + \frac{1}{n^2} \right]$$

$$\lim_{n \to \infty} U_n = \frac{b^4}{4}$$

Kubische Funktion



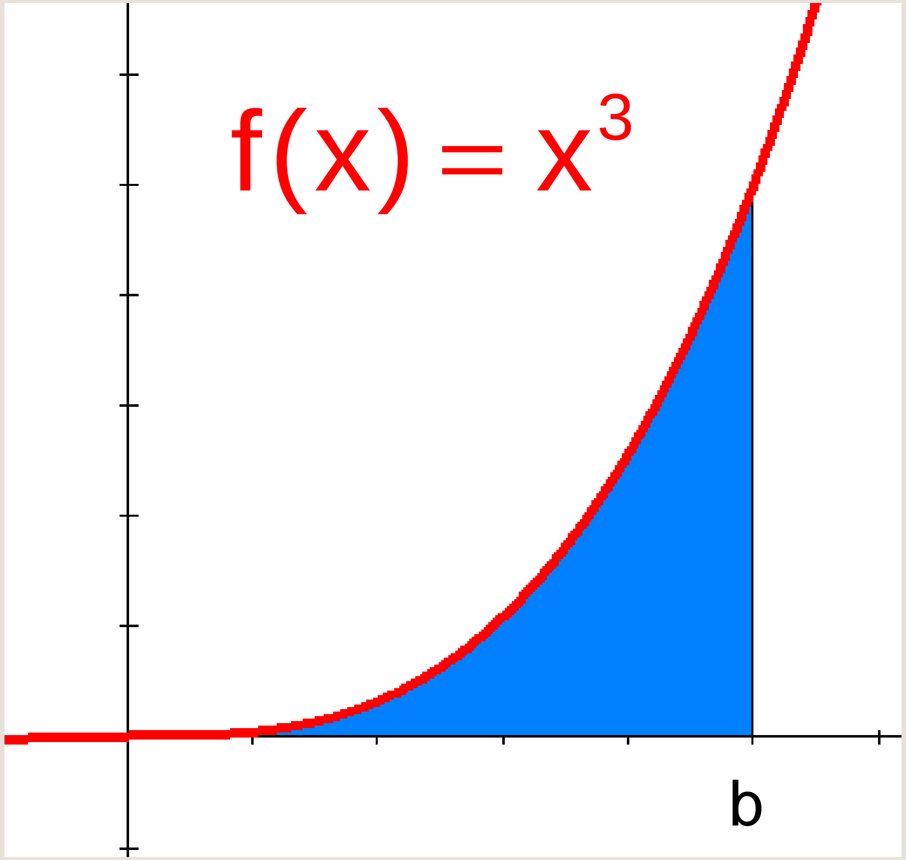
$$O_n = \frac{b}{n} \cdot \left[f(1 \cdot \frac{b}{n}) + f(2 \cdot \frac{b}{n}) + \ldots + f(n \cdot \frac{b}{n}) \right]$$

$$O_n = \frac{b^4}{n^4} \cdot \left[1^3 + 2^3 + \ldots + (n-1)^3 + n^3 \right]$$

$$1^3 + 2^3 + \ldots + n^3 = \left(\frac{n \cdot (n+1)}{2} \right)^2 \Rightarrow O_n = \frac{b^4}{n^4} \cdot \left[\frac{n^2 \cdot (n+1)^2}{4} \right] = \frac{b^4}{4} \cdot \left[1 + \frac{2}{n} + \frac{1}{n^2} \right]$$

$$\lim_{n \to \infty} O_n = \frac{b^4}{4}$$

Kubische Funktion

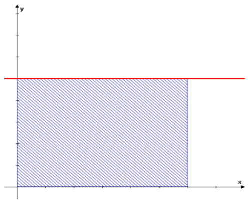
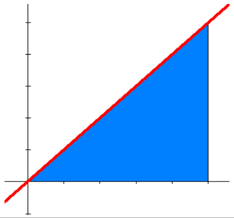
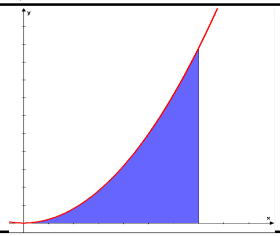
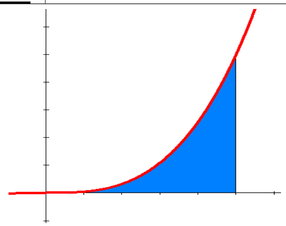


$$\lim_{n\to\infty} O_n = \frac{b^4}{4}$$

$$\lim_{n\to\infty} U_n = \frac{b^4}{4}$$

$$A_o^b = \frac{b^4}{4}$$

Übersicht

	$f(x) = x^0$	$A_o^b = \frac{b^1}{1}$
	$f(x) = x^1$	$A_o^b = \frac{b^2}{2}$
	$f(x) = x^2$	$A_o^b = \frac{b^3}{3}$
	$f(x) = x^3$	$A_o^b = \frac{b^4}{4}$
	$f(x) = x^n$	$A_o^b = \frac{b^{n+1}}{n+1}$

$n \in \mathbb{N}$